

Individual Project 3
 $\int_1^x \frac{1}{t} dt$ Should Equal $\ln x$
Due Wednesday, Feb. 10

Project Assignment: Notice that for all functions $f(t) = t^r$ the antiderivatives are of the form

$$\int t^r dt = \frac{t^{r+1}}{r+1} + c$$

with the exception of one case when $r = -1$. It seems confusing that

$$\int \frac{1}{x} dx = \ln |x| + c$$

but in this project you will show that this is actually consistent with the other results. To do this you will use properties of logarithms.

1. Let $F(r) = \int_1^2 t^r dt$, with r a real number not equal to -1. Evaluate $F(r)$ for several values of r close to -1. Create a table of these values and include them in your report.
2. Does there appear to be a value for the limit $\lim_{r \rightarrow -1} F(r)$? What is the value of the limit?
3. Repeat the process in 1 and 2 with the upper limit changed from 2 to 3.
4. Repeat this again with the upper limit changed to 4.
5. Repeat this a fourth time with the upper limit changed to 10.
6. Let b be a fixed positive number. For r a real number not equal to -1, redefine the function F by $F(r) = \int_1^b t^r dt$. Find a simple formula for $F(r)$.
7. Note that F is a continuous function except at $r = -1$ where it is undefined. Estimate the limit $\lim_{r \rightarrow -1} F(r)$ for several different values of b , either by graphing $F(r)$ or by plugging in values of r close to -1. Compare the results to $\ln b$.
8. How should we define $F(-1)$ so that $F(r)$ is continuous at -1?
9. Explain how $\int_1^x \frac{1}{t} dt$ is consistent with all the other antiderivatives of t^r when $r \neq -1$.

You should print your report and turn it in by the due date.